

# The Computational Basis of Locality in Syntactic Agreement

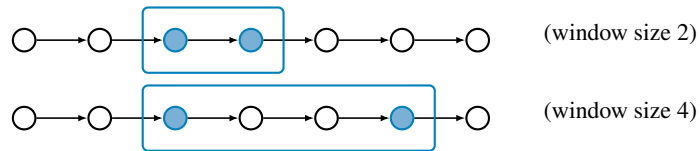
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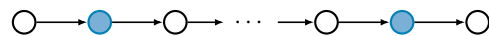
## 1 Introduction

### ■ What does it mean to be local? ■

Local → finitely bounded



Long-distance (non-local) → no finite bound



### ■ Not just local, *strictly local* ■

**Strictly local (SL):** constraints on substrings (or subtrees) of fixed size (Rogers et al. 2013; Rogers 1997).

#### Ex. Local assimilation

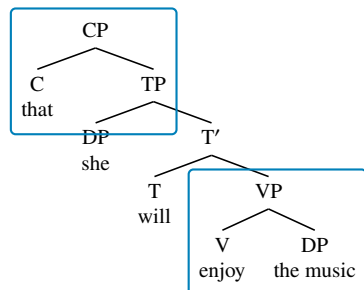
✓ a m p a p a n d a

✗ a n p a p a n d a

✗ a m p a p a m d a

Constraints:  
\*np, \*nb, \*mt, \*md, ...

#### Ex. Category selection



Constraints:  
\* CP    \* VP ...  
C    VP    V    DP  
that    laugh

This presentation adapts an MG derivation tree-based analysis to mainstream Minimalism. See Hanson (2023a) for a more formal presentation (<https://www.kennethhanson.net/files/hanson-nyubb2023-agreement-slides.pdf>).

The full structure is well-formed iff all substructures are well-formed. The contents of different window positions may not be compared.

### ■ Adding in non-locality ■

**Tier-based strictly local (TSL) patterns:** like SL, but *irrelevant elements are ignored* (Heinz et al. 2011; Lambert and Rogers 2020).

#### Ex. Sibilant harmony (Heinz 2018)

✓ p i s o t o n o s i k i w a t

✗ p i s o t o n o f i k i w a t

Visible elements: s, f  
Constraints: \*sf, \*fs

#### Ex. $\phi$ -agreement (this talk)

✓ There **seem**<sub>PL</sub> [TP to be some problems<sub>PL</sub> with this theory].  
% There **seems**<sub>SG</sub> [TP to be some problems<sub>PL</sub> with this theory].

Visible elements: finite T, all D  
Constraints: \*SG · PL, \*PL · SG

(To be revised.)

### ■ Overview ■

**Main claim:** Long-distance linguistic dependencies are predominantly TSL with a window size of two – they are **tier-based strictly 2-local (TSL-2)**.

- Phonotactics (McMullin 2016; McMullin and Hansson 2016)
- Morphotactics (Aksénova et al. 2016)
- Movement (Graf 2018; Graf 2022)
- Case assignment (Hanson 2023b)
- **Agreement (this work)**

**Focus for today:** TSL-2 provides a **unified model of locality** based on a **moving window of visibility**.

- Encompasses relativized minimality (Rizzi 1990; Rizzi 2013), selective opacity (Keine 2019), case discrimination (Bobaljik 2008), and more.
- Derives (one type of) myopia in grammar from computational considerations (Rogers et al. 2013; Lambert et al. 2021).

### ■ Roadmap ■

1. TSL patterns and their properties
2. A TSL model of agreement
3. Consequences for locality
4. Typological variation, parallels across domains
5. Strengths and limitations of the model

## 2 TSL patterns

### ■ What is a TSL pattern? ■

1. Ignore the irrelevant items and treat the rest as if adjacent
2. All constraints must be stated within a fixed-size moving window

### Example: Vowel harmony

i/u/o obey front-back harmony, *e* is transparent/neutral, *a* is opaque

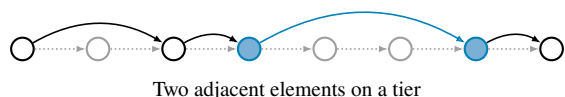
All elements:	k, b, l, i, e, u, o, a
Tier elements:	i, u, o, a
Invisible elements:	k, b, l, e
Window size:	2
Constraints:	*iu, *io, *oi, *ui

Word	Tier	
kubulo	uuo	✓
kibilo	iio	✗
kubelo	uuo	✓
kibelo	io	✗
kubalo	uao	✓
kibalo	iao	✓

See Appendix A.1 for another example and a formal definition.

### ■ More about TSL ■

- Inspired by but distinct from autosegmental phonology (Goldsmith 1976)
- Special relational structure combined with very weak constraint logic (cf. Lambert and Rogers 2020; Lambert et al. 2021; Lambert 2023)
- Related: ITSL and OTSL functions model a wide range of phonological maps (Burness et al. 2021)



## 3 A TSL Model of Agreement

### ■ Setup ■

#### Assumptions:

- Bare phrase structure, feature-driven selection, movement, some method of case assignment
- Agreement between elements with initially unvalued features (probes) and elements which provide those values (goals)<sup>1</sup>

**Question:** What are the possible arrangements of probes and goals for agreement?

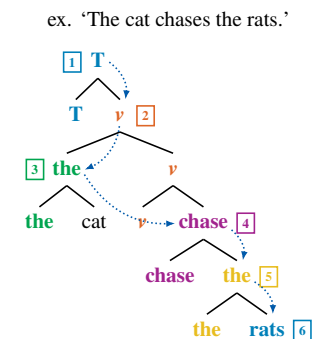
**Answer:** They are TSL-2 constraints on the “search path” of the probe.

<sup>1</sup>I use the term **agreement** rather than **Agree** to indicate that we are not dealing with other phenomena that are sometimes subsumed under Agree. I retain the terms probe and goal for convenience.

### ■ The search path ■

I assume that the search path follows the **derivational command (d-command)** relation (Graf and Shafiei 2019).

- Head < Spec < Comp
- d-command order ≈ height of XP  
≈ order of last merge  
≈ reverse order of selection
- Projections of a head are not distinguished.
- At each branching point, follow the complement spine (Graf and De Santo 2019).



See Appendix A.3 for how this works using derivation trees.

### ■ The TSL analysis ■

**General principle:** a probe must be immediately followed by its goal on a tier projected from the search path (and vice versa).

**Notation:**  $p\phi$  = probe  $g\phi$  = actual goal  $\phi$  = other potential goal

### Example: (canonical) subject-verb agreement

Tier elements: All agreeing elements (T/D) and blockers (C)

Constraints:  $*T_{[p\phi]} \cdot D_{[\phi]}$ ,  $*T_{[p\phi]} \cdot C$ ,  $*D \cdot D_{[g\phi]}$ ,  $*D_{[g\phi]} \cdot D_{[g\phi]}$ , ...

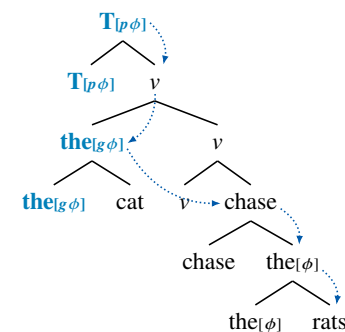
### ■ The TSL analysis – example ■

The cat chases the rats.

Path:  $T_{[p\phi]} \cdot v \cdot D_{[g\phi]} \cdot v \cdot D_{[\phi]} \cdot N$

Tier:  $T_{[p\phi]} \cdot D_{[g\phi]} \cdot D_{[\phi]}$

Violations: n/a



For simplicity, we substitute most items with their category labels in the path and tier projection.

## 4 Consequences for locality

### ■ Consequences for locality ■

- **Blocking:** if another element intervenes on the tier, agreement is blocked, regardless of whether the blocker itself can agree.
  - relativized minimality (Rizzi 1990; Rizzi 2013)

- domain-based blocking

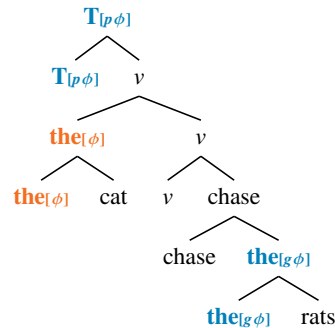
• **Invisibility:** if a DP is omitted from the tier, strict minimality may be violated.

- agreement across *there*

- case-sensitive agreement (Bobaljik 2008; Preminger 2014)

■ **Minimality** ■

If another potential goal intervenes on the tier, agreement is blocked.



\* The cat **chase** the rats.

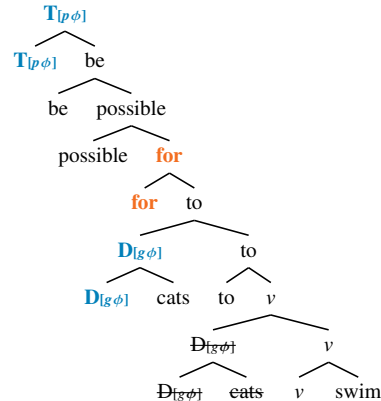
Path:  $T_{[p\phi]} \cdot v \cdot D[\phi] \cdot V \cdot D[g\phi] \cdot N$

Tier:  $T_{[p\phi]} \cdot D[\phi] \cdot D[g\phi]$

Violations: \*  $T_{[p\phi]} \cdot D[\phi]$ , \*  $D[\phi] \cdot D[g\phi]$

■ **Domain-based blocking** ■

If a non-agreeing element is projected on the tier, agreement is likewise blocked.



\* It **are** possible for cats to swim.

cf. It **is** possible. . .

Path:  $T_{[p\phi]} \cdot v \cdot V \cdot C \cdot T \cdot D[g\phi] \cdot v \cdot D_{[g\phi]} \cdot V$

Tier:  $T_{[p\phi]} \cdot C \cdot D[g\phi]$

Violations: \*  $T_{[p\phi]} \cdot C$ , \*  $C \cdot D[g\phi]$

Assume for the sake of demonstration that expletive "it" is inserted late and does not agree.

■ **Invisibility** ■

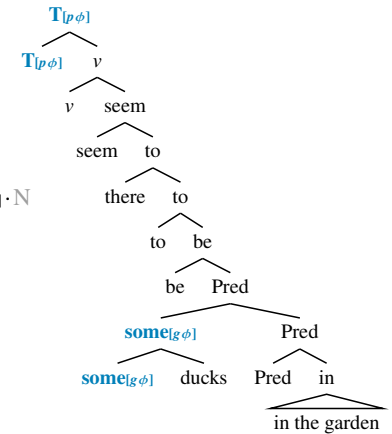
If a DP is omitted from the tier, strict minimality may be violated.

There **seem** to be some ducks in the garden.

Path:  $T_{[p\phi]} \cdot v \cdot V \cdot T \cdot there \cdot v \cdot Pred \cdot D[g\phi] \cdot P \cdot D[\phi] \cdot N$

Tier:  $T_{[p\phi]} \cdot D[g\phi] \cdot D[\phi]$

Violations: n/a



We can handle optional default agreement in several ways. Ask me if you are interested.

■ **Case-sensitive agreement** ■

In Hindi, the verb agrees with the closest *nominative* argument, which may not be the subject.

(1) Hindi verbal agreement ignores ergatives (Mahajan 1990)

a. Raam roTii khaataa thaa.  
Raam.M.NOM bread.F.NOM eat.IPFV.M be.PST.M

'Raam ate bread (habitually).'

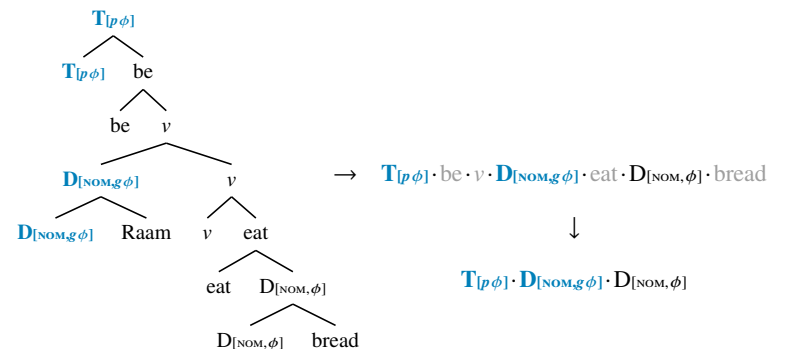
b. Raam-ne roTii khaayii.  
Raam.M-ERG bread.F.NOM eat.PFV.F

'Raam ate bread.'

**Analysis:** Project D only if nominative. Tier constraints are unchanged.

■ **Case-sensitive agreement (2)** ■

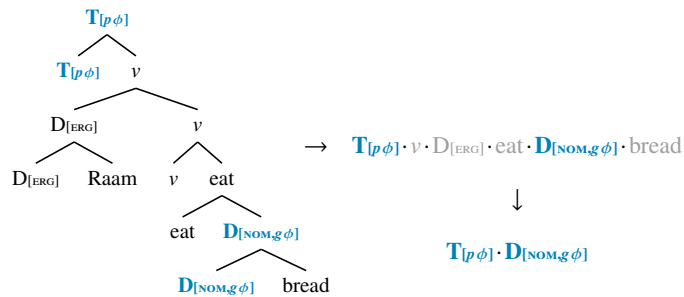
'Raam ate bread (habitually).' (Nominative subject, subject agrees)



We ignore agreement on the non-finite verb for simplicity.

■ **Case-sensitive agreement (3)** ■

‘Raam ate bread.’ (Ergative subject, object agrees)



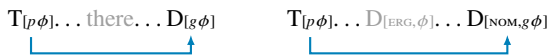
■ **Locality – interim summary** ■

Locality phenomena derive from TSL with a window of size two, a.k.a. **TSL-2**.

- Blocking: some (non-)agreeing element intervenes on the tier



- Invisibility: hypothetical goal does not appear on tier



■ **The importance of the finite window** ■

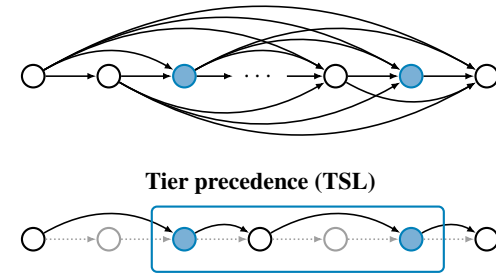
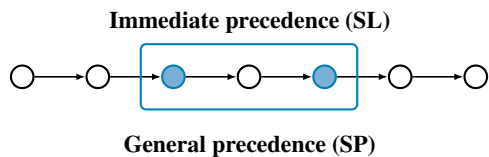
- Neither tiers nor the finite window alone are adequate.
  - Tiers provide relativized locality.
  - Keeping all constraints within the moving window limits the power of the system.
- Other combinations of locality profile and constraint logic are either too powerful, too weak, or both.

■ **Limits on structural configurations** ■

TSL computations can relate elements at a distance, but are otherwise severely restricted in what they can do.

- No arbitrary logic — “a DP can A-move out of a finite CP, but only if there is A'-movement within some (other) CP in the sentence”
- No counting — “up to three reflexive pronouns may occur in a sentence if each obeys the Binding Theory”

■ **Three models of locality** ■



Name of formal class combining locality model with logic of banned substructures in parentheses.

■ **Three models of locality (2)** ■

- SL (immediate precedence) can handle local spreading.
- SP (general precedence) can handle unbounded processes, but can't handle blockers.
- Only TSL (tier precedence) can handle unbounded processes with blocking.

**5 Typological variation**

■ **Variation in visibility** ■

- Not all of invisibility and blocking can be analyzed purely in terms of relativized minimality
- Sometimes, even items which almost certainly possess the relevant features are invisible nonetheless
- Conversely, irrelevant items may block agreement
- Both types of exception are subject to variation across languages and dependencies

■ **Case discrimination, revisited** ■

Ergatives are not invisible in Nepali (though datives are).

(2) Agreement with ergative in Nepali (Coon and Parker 2019)

- Maile yas pasal-mā patrikaā kin-ē.  
**1sg.ERG** DEM store-LOC newspaper.ABS buy-**1sg**  
 ‘I bought the newspaper in this store.’
- Ma thag-ī-ē.  
**1sg.ABS** cheat-PASS-**1sg**  
 ‘I was cheated.’

No problem! We project  $D_{[NOM]}$  and  $D_{[ERG]}$  but not  $D_{[DAT]}$ .

■ **Selective opacity** ■

“A particular structure may allow one movement [or agreement] type to proceed out of it but at the same time block other types of extraction [agreement].” — Keine (2019)

	Probe location	Size of clause		
		CP (finite)	TP (nonfinite)	vP (nonfinite)
$\phi$ -agreement	T	*	*	✓
A-movement	T	*	*	✓
Wh-licensing	C	*	✓	✓
A'-movement	C	✓	✓	✓

Selective opacity in Hindi (adapted from Keine 2019)

### ■ Formal vs substantive constraints ■

- Case visibility hierarchy (Bobaljik 2008):  
Nom > Acc/Erg > Obliques
- Height-locality connection (Keine 2019): The possible horizons for a (nonvacuous) probe on a head X which is a projection of Y **exclude** all extended projections of Y below X  
e.g. T cannot be a horizon for a probe on C
- We can encode the attested patterns in a TSL-2 grammar, but the implicational hierarchy itself requires a separate explanation.

### ■ Parameters for variation ■

The parameters for TSL-2 (tier elements and constraints) correspond neatly to variation in long-distance dependencies.

1. Visibility — which elements are relevant and which are ignored?
  - Case-sensitive agreement (cf. Bobaljik 2008; Preminger 2014)
  - Probe horizons (Keine 2019)
2. Iteration — if you allow AB and BB, then you get ABB, AB BB, etc.
  - Case/gender/number concord
3. Directionality — do we ban AB or BA?
  - Upward/downward agreement (cf. Chomsky 2000; Zeijlstra 2012; Carstens 2016)

For #2 & #3, see Appendix A.2

### ■ What else is TSL-2? ■

Phenomenon	One line summary
Defective intervention*	Some DPs project even if they are never $g\phi$
A'-agreement (Van Urk 2015)*	Only project DPs with a certain A' feature
Omnivorous number	Only project DPs with [PL], not [SG]
Upward C agr. (Diercks 2013)*	C probes up, only project DPs that EPP-move
Default agreement*	Allow lone $p\phi$ under limited circumstances
Interaction/Satisfaction (Deal 2015)*	Allow multiple $g\phi$ under limited circumstances
Parasitic agreement (Bhatt 2005)	Allow parasitic elements btw. $p\phi$ and $g\phi$
Independent subfeatures of $\phi$	Each probe gets its own tier/constraints

Also: many movement (Graf 2022) and case patterns (Vu et al. 2019; Hanson 2023b), though these analyses use a different tier-based model.

\*See Hanson (2023a) and Hanson (2024a) for details.

### ■ What isn't TSL? ■

Some linguistic patterns are not TSL, but SS-TSL (structure-sensitive TSL):

- Some long-distance harmony (De Santo and Graf 2019; Graf and Mayer 2018)
- Some tone patterns (e.g. unbounded tone plateauing)
- Some binding rules (Graf and Shafiei 2019)

## 6 Parallels across domains

### ■ Parallels across domains ■

Parameter	$\phi$ -agreement	Wh-movement	Vowel harmony
Participants	Probe and most DPs	Probe and Wh-DPs	Most vowels
Invisible	Non-DPs, some DPs	Non-Wh elements	Consonants, some V
Blockers	Finite C, some DPs	Certain islands	Some vowels
Directionality	Downward/upward	???	Progressive/regressive
Chaining	Concord/no concord	Wh-agreement <sup>2</sup>	Spreading/"icy targets"

### ■ Locality and typology ■

Type	Class	Example	Visible Cs
Unbounded	TSL-2	Aari	Only sibilants
LD w/ blocking	TSL-2	Slovenian	All coronals
Transvocalic	TSL-2	Koyra	All consonants
At most 1 C intervenes	TSL-3	Unattested	—
Exactly 1 C intervenes	TSL-3	Unattested	—
At least 1 C intervenes	TLT <sup>3</sup>	Unattested	—

Typology of consonant (dis)-harmony (adapted from McMullin and Hansson 2016)

### ■ Locality and typology (2) ■

Non-TSL-2 and unattested island types (Graf 2022):

- Gang-up islands: A mover can escape  $n$  islands, but not  $n + 1$ .
- Configurational islands: XP is an island iff it is inside an embedded clause.
- Cowardly islands: XP is an island iff there are at least  $n$  XPs in the same clause.
- Narcissist islands: XP is an island iff there are no other XPs in the same clause.
- Rationed island effects: At most  $n$  phrases per clause can be an island.
- Discerning islands: XP is an island only for movers that contain a PP.

### ■ Locality and typology (3) ■

Non-TSL-2 and unattested subject-verb agreement types:

- Matrix T agrees with the embedded subject, and embedded T with the matrix subject.
- T agrees with the subject only in a ditransitive clause, otherwise default agreement is required.

<sup>2</sup>See Graf (2022).

<sup>3</sup>Maybe also SS-TSL, specifically OTSL [K.H.].

- T agrees with the subject unless there is a temporal adjunct, in which case it agrees with the object.
- When finite clauses are coordinated, agreement occurs in exactly one of them.
- Only DPs which contain a relative clause which contains two PPs can agree.

## 7 Strengths and limitations of the model

### ■ Advantages of the model ■

- Clear separation of concerns:
  - Structural representation
  - Computations over said structure
  - Substance of elements of structure
- Restrictive, independently rooted in well-understood mathematics
  - Subregular language hierarchy, relativized adjacency, etc.
- Agnostic to many analytical choices
  - e.g. Basic functional hierarchy vs cartographic hierarchy

### ■ Limitations of the model ■

Puzzles for the path-based approach:

- What to do about violations of c-command (e.g. sub-command)?
- How to handle exceptions to the complement spine generalization?

What the formal model does not tell us:

- Why does case matter for  $\phi$ -agreement? Why should nominatives always be visible, ergatives sometimes visible, and datives usually invisible?
- Why do probes seem to look downward more often than upward?
- How do children identify the visible elements and constraints for each dependency? (see Hanson 2024b; Belth 2023)

## 8 Conclusion

### ■ Summary ■

- A wide range of facts about the locality profiles of linguistic dependencies are explained if they are TSL with a window size of 2.
- From this perspective, agreement turns out to be especially similar to phonological harmony — perhaps because both are feature-matching phenomena.
- Most of the logical possibilities of the model are realized within a single phenomenon — this is not necessarily expected!

### ■ Some open questions ■

- Do we ever need a window size larger than 2?
- Are there agreement patterns that are not TSL under any reasonable analysis?
- How far can we take the parallel with harmony in phonology?

### ■ Takeaways ■

- Computational approaches to linguistic analysis reveal insights that might otherwise not be obvious.
- In other cases, they provide independent support to conclusions reached in other ways (e.g. visibility is parameterized).
- A clear understanding of the formal patterns can help us understand other aspects of linguistic structure.

### ■ Acknowledgments ■

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## A Extras

### A.1 Even more on TSL

#### ■ Extra example: Sibilant harmony ■

Sibilants match in anteriority, *t* blocks harmony, other C’s transparent  
(based on Slovenian)

	Word	Tier	
All elements:	{s, f, t, k, a}		
Tier elements:	{s, f, t}		
Constraints:	{*sf, *fs}		
	s a s a s a	s s s	✓
	s a s a f a	s s f	✗
	s a k a s a	s s	✓
	s a k a f a	s f	✗
	s a t a s a	s t s	✓
	s a t a f a	s t f	✓

#### ■ TSL string languages – formal definition ■

In a **tier-based strictly *k*-local (TSL-*k*)** language, a string is well-formed iff its **tier projection** does not contain any forbidden substrings of some length *k*.

- $\Sigma$  = “alphabet” = set of all symbols
- $T$  = “tier alphabet” = set of visible symbols
- $G$  = “grammar” = forbidden substrings
- The tier projection is obtained by deleting all non-tier elements and concatenating the remaining elements.

**Example: vowel harmony (redux)**

$\Sigma = \{k, b, l, i, e, u, o, a\}$
$T = \{i, u, o, a\}$
$k = 2$
$G = \{iu, *ui, *io, *oi\}$

String	Tier projection	Substrings	
kubulo	uuo	{uu,uo}	✓
kibilo	iio	{ii,io}	✗
kubelo	uuo	{uo}	✓
kibelo	ioo	{io}	✗
kubalo	uao	{ua,ao}	✓
kibalo	iao	{ia,ao}	✓

### A.2 More agreement patterns

#### ■ Concord in the DP ■

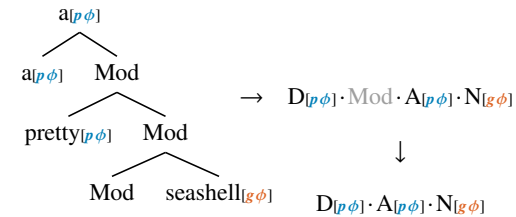
To allow for iterated agreement, just permit  $p\phi \cdot p\phi$ .

- (3) Gender concord in German  
Ich habe [eine hübsche Muschel] gefunden.  
I have [a.F pretty.F seashell.F] found  
‘I found a pretty seashell.’

**Analysis:** Ignore Mod on the tier, permit  $D_{[p\phi]} \cdot A_{[p\phi]}$  and  $A_{[p\phi]} \cdot A_{[p\phi]}$ .

#### ■ Concord in the DP (2) ■

**Analysis:** Ignore Mod on the tier, permit  $D_{[p\phi]} \cdot A_{[p\phi]}$  and  $A_{[p\phi]} \cdot A_{[p\phi]}$ .



The Mod head is not crucial. If direct adjunction is used, then the pattern is local: the tier contains everything.

#### ■ Upward agreement ■

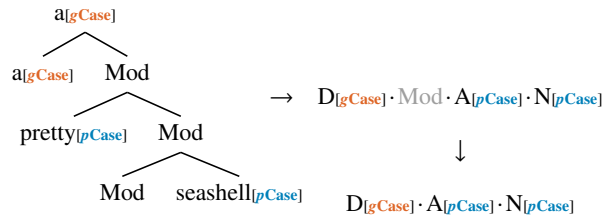
If the constraints are mirrored, then the direction of agreement is reversed.

- (4) Case concord in German  
Ich habe [eine hübsche Muschel] gefunden.  
I have [a.ACC pretty.ACC seashell.ACC] found

**Analysis:** allow  $D_{[gCase]} \cdot A_{[pCase]}$  instead of  $D_{[pCase]} \cdot A_{[gCase]}$ , etc.

#### ■ Upward agreement (2) ■

**Analysis:** allow  $D_{[gCase]} \cdot A_{[pCase]}$  instead of  $D_{[pCase]} \cdot A_{[gCase]}$ , etc.



We can handle definiteness agreement on the adjective (ignored here) in the same way.

■ **What does it mean to probe upward?** ■

- In the MG derivation tree formalism (Graf and Shafiei 2019), we have a static representation of the entire derivation, so there is no problem.
- In a bottom-up Minimalist derivation, it is not obvious what it means for a probe to search upward. Some possibilities:
  - Let valued features search downward for unvalued features (Adger 2003)
  - Let Agree be upward, triggered once the relevant items are merged (Zeijlstra 2012)
  - Replace the search metaphor with the sliding window metaphor (my suggestion)

■ **Interaction and satisfaction (Deal 2015)** ■

**Basic idea:** Every probe has features that it interacts with (interaction set) and features that cause probing to stop (satisfaction set).

**TSL analysis:** Probe may be followed by zero or more interacting heads, followed by one that satisfies it.

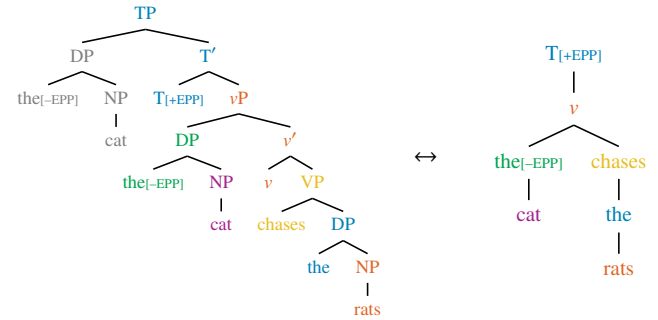
Formally (but simplified):

- Let P denote a head with the probe.
- Let I denote a head with features in the interaction set *but not* the satisfaction set.
- Let S denote a head with features in the satisfaction set.
- Allowed pairs: P · S, P · I, I · I, I · S, ...
- Banned pairs: S · I

**A.3 A more formal syntactic model**

■ **MG derivation trees** ■

- All nodes appear in base position.
- The rightmost child of a node is its complement; others are specifiers.
- Movement is indicated using feature diacritics.

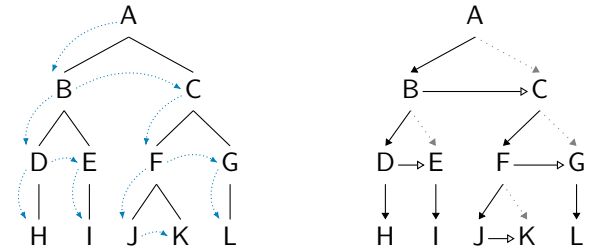


See Graf and Kostyszyn (2021) for a formal definition. Related: Brody (2000).

■ **Command strings** ■

A **command string** (c-string) is a derivational ordering of nodes.

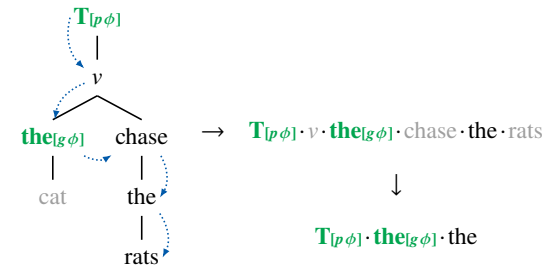
- There is a c-string from the root to each node.
- Among each head and its arguments: Head < Specifier < Complement.



See Graf and Shafiei (2019) for details.

■ **Tiers over command strings** ■

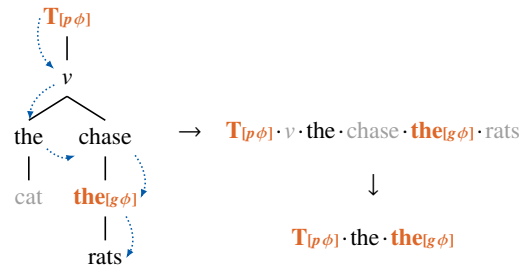
✓ The cat **chases** the rats. (subject agreement)



■ **Tiers over command strings (2)** ■

✗ The cat **chase** the rats. (object agreement)

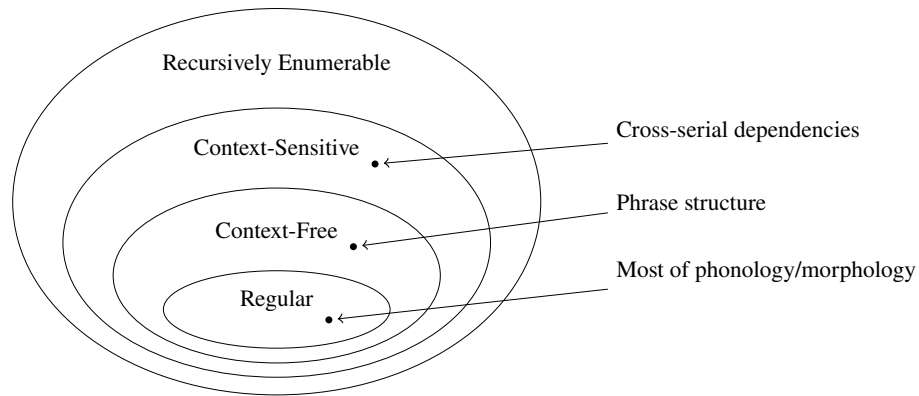




#### A.4 Computational background

##### ■ The Chomsky Hierarchy ■

Syntax is “mildly context sensitive” when analyzed over surface strings. It becomes subregular when analyzed over derivation trees.



##### ■ The Subregular Hierarchy ■

